Designing The Weatherbell: A Collaboration with Marcus Vergette

17 Bower St
Northcote
Australia, 3070

Dr Neil McLachlan
Tuned Percussion Instruments
5/1/2007

Introduction

I have a doctorate in physical sciences but have worked professionally in music and musical instrument design since 1986. Over the last decade I have developed extensive experience in designing musical bells. This work featured the first design of a bell with harmonic overtones and unambiguous pitch.

Marcus approached me in late 2005 about designing a bell that could be sounded by the action of waves. After a series of emails over the next 6 months we arrived at a design concept of a bell that could produce multiple pitches when struck at different locations.

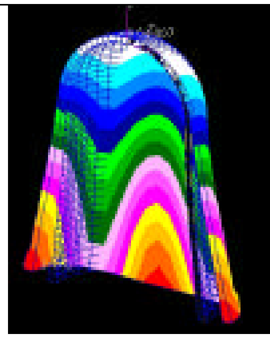
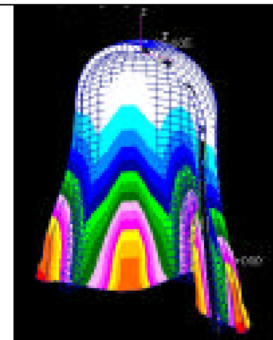
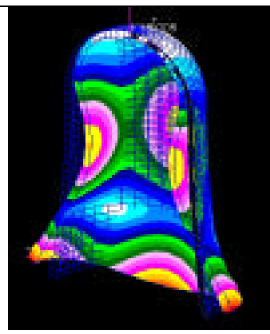
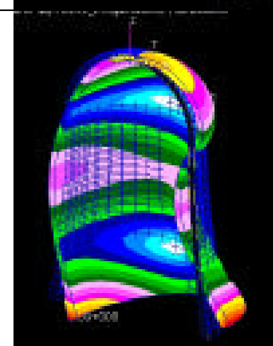
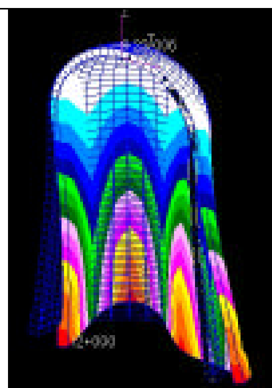
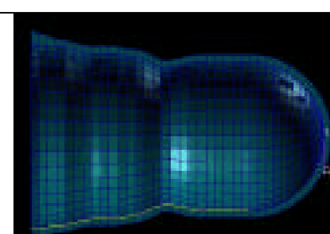
This required designing a bell where the maxima of vibration of groups of modes occurred in distinct regions of the bell profile, and that these groups of modes be tuned to harmonic relationships within each group and some musical relationship between the groups. Musical relationships can most easily be described as simple frequency ratios such as 3/2 (a just 5th) or 5/3 (a just 6th). Chromatic tuning are small deviations from these ratios. From previous experience I knew that a generally cylindrical form would best separate modes with different numbers of nodal rings (see later for mode shape details).

With the support of seed funding some preliminary explorations were undertaken in Melbourne before visiting Marcus in Devon for 3 days in September 2006. During this visit the design was further developed and the ‘hourbell’ shape was proposed by Marcus to introduce more possible tones to the design. The design was then further refined upon my return to Melbourne. The ‘hourbell’ bell can produce four distinct pitches by being struck near the rim or centre of the bell at each end.

In the following section the various mode shapes and frequencies of bell overtones and an example tuning process are described. In general vibratory modes that efficiently radiate sound can be classified by the number of nodal lines that run vertically through the centre of the bell (n) and nodal rings around the circumference (m). Each vibratory mode is given the identifier n,m. The bell is stationary along a nodal line and the vibration changes phase across a nodal line.

Design Data

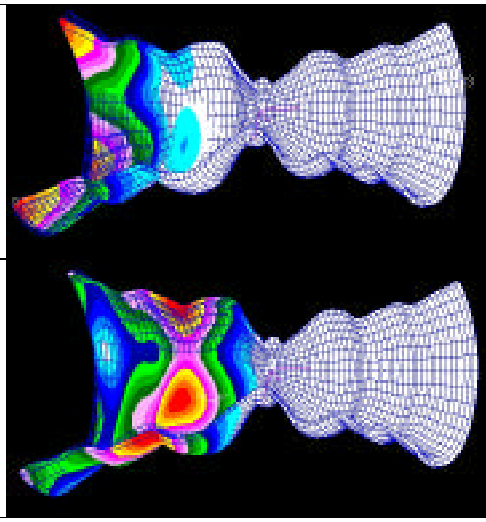
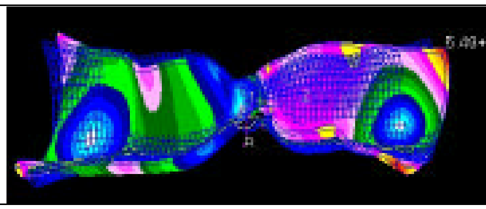
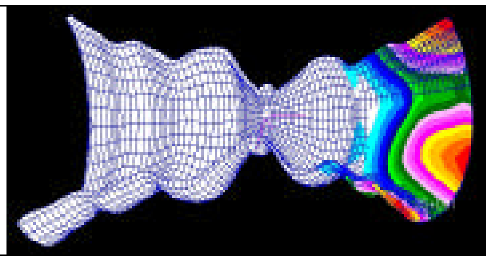
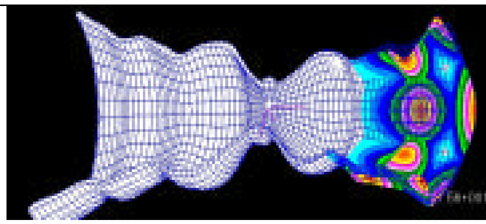
The following table shows the overtones of slightly conical cylindrical bell with a thickened rim. This profile is the result of a series of design studies to find an optimal starting shape.

Mode type	Freq (Hz) ratio	Mode shape	Mode type	Freq (Hz) ratio	Mode shape
2,0	220 1		4,0	923 4.20	
2,1	395 1.80		T2	1017 4.62	
3,0	507 2.31		3,2	1241 5.64	
3,1	802 3.65				 Geometry after the 2nd optimisation.

Red colours show the region of the displacement maximum for each mode shape. The unperturbed geometry is the blue grid. Note that striking the bell 1/2 way up the wall will excite the n,1 and n,2 modes much more than the n,0 modes, whereas striking the bell about 1/8 way up (on the n,1 nodal ring) will only excite the n,0 modes.

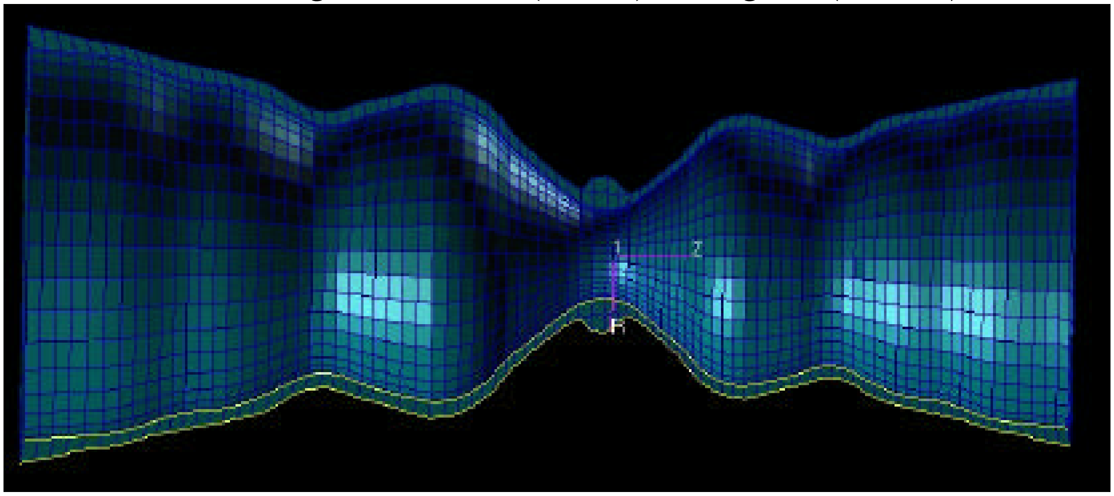
The shape was optimised to align overtones to harmonic ratios of 2 pitches with a harmonious relationship between them. Rescaling the bell and joining the two bells end-to-end creates an hour-bell shape able to produce 4 discrete pitches.

Hourbell1 Data

Large bell			
	Mode type	Freq (Hz)	Example Mode shapes
1	2,0	204	
2	2,1	441	
3	3,0	483	
4	T3	766	
5	3,1	788	
6	4,0	875	
Small bell			
	Mode type	Freq (Hz)	
1	2,0	217	
2	3,0	531	
3	2,1	558	
4	3,1	845	

To show how the design process was undertaken the last set of design iterations to arrive at the first prototype design are detailed below. There were two other design iteration sets that failed to arrive at sufficiently close solutions and were abandoned.

Hourbell2 – after large 2,1 to 350 Hz (mode 4) and large 4,0 (mode 10) to 800 Hz

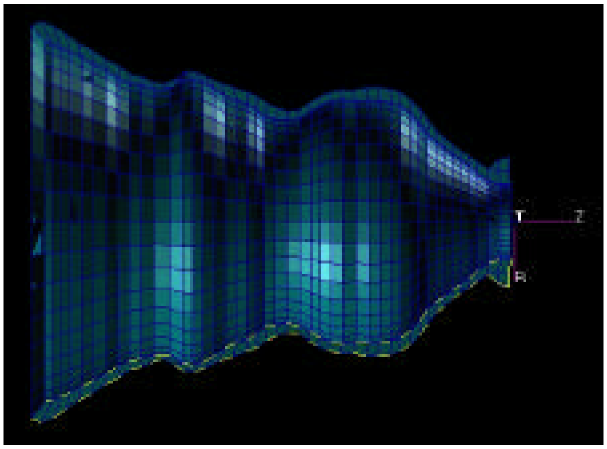


Below are the optimisation targets for this optimisation run ($ lines – disregarded):

```
response(strategy=c3,fmin=100)
$   mode(var=f2,target=200,tol=1)
    mode(var=f4,target=350,tol=3)
$   mode(var=f6,target=400,tol=4)
    mode(var=f10,target=800,tol=8)
$   mode(var=f12,target=1050,tol=10)
```

Note, ‘tol’ defines the design tolerance for each optimisation target.

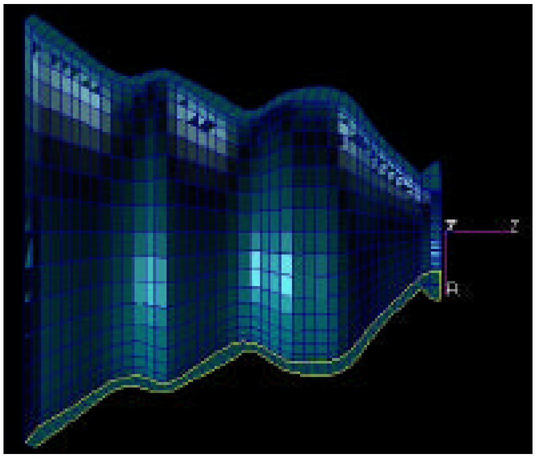
Hourbell3 (large bell only so new mode definitions) – after 2,1 to 350 Hz, 3,0 to 400 Hz and 4,0 to 600 Hz. Running only _ the hourbell was more efficient for large geometry changes as long as the correct ‘boundary conditions’ are applied to the model.



Optimisation targets:

response(strategy=c3,fmin=100)
  mode(var=f1,target=200,tol=1)
  mode(var=f2,target=350,tol=3)
  mode(var=f3,target=400,tol=4)
  mode(var=f4,target=600,tol=6)

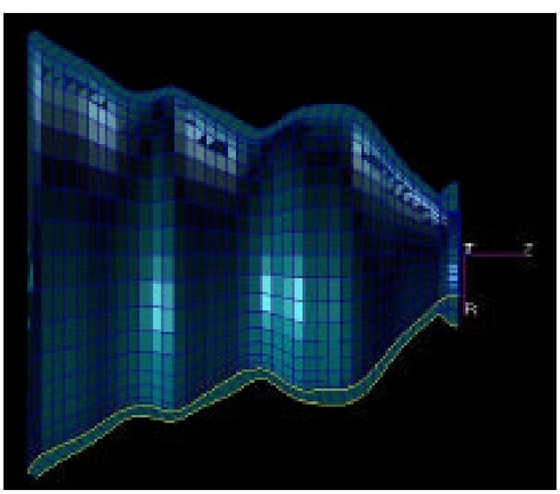
Hourbell4 – after 2,0 to 220 Hz, 2,1 to 385 Hz, 3,0 to 440 Hz and 4,0 to 660 Hz



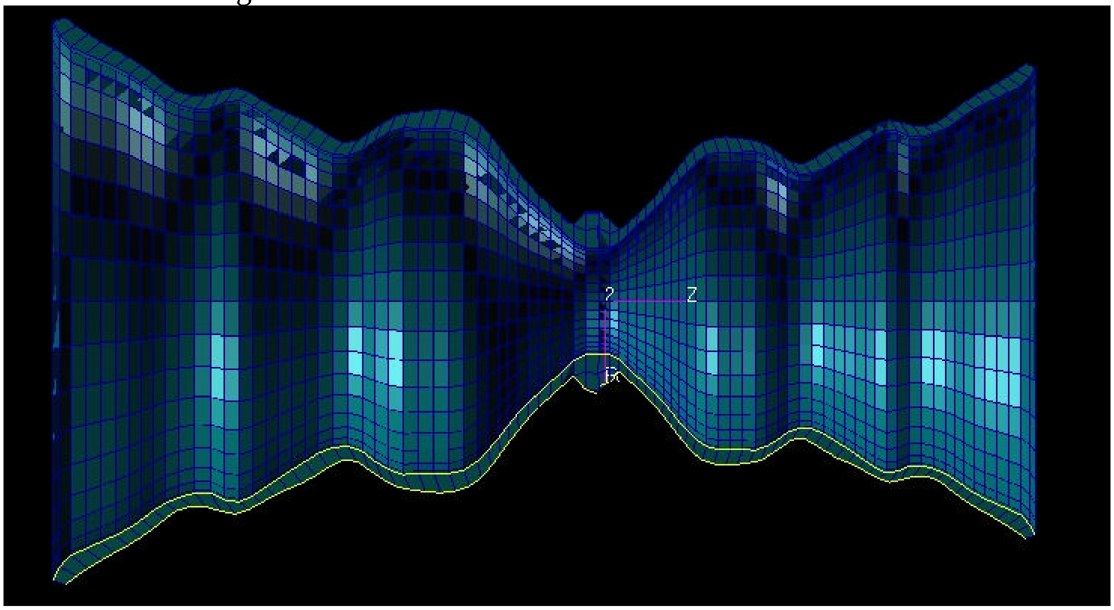
Optimisation targets:

response(strategy=c3,fmin=100)
  mode(var=f1,target=220,tol=1)
  mode(var=f2,target=385,tol=3)
  mode(var=f3,target=440,tol=4)
  mode(var=f4,target=660,tol=4)
$ mode(var=f10,target=800,tol=8)
$ mode(var=f12,target=1050,tol=10)

Hourbell5 – rerun of above with tighter tolerances.



Hourbell6 – full bell again but ignoring the results for the small bell in the optimisation. The large bell is then rescaled to replace mistuned small bell. The results of this design iteration are shown in the table below.



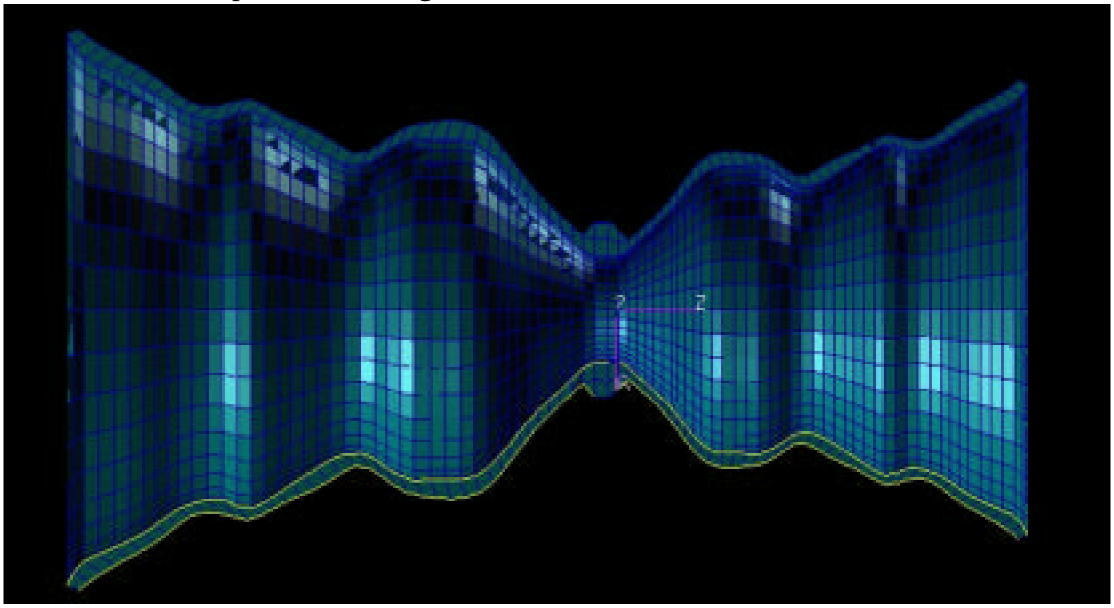
Optimisation targets:

```
response(strategy=c3,fmin=100)
   mode(var=f1,target=220,tol=1)
   mode(var=f2,target=262,tol=3)
   mode(var=f3,target=392,tol=4)
   mode(var=f9,target=660,tol=4)
```

Hourbell6 data

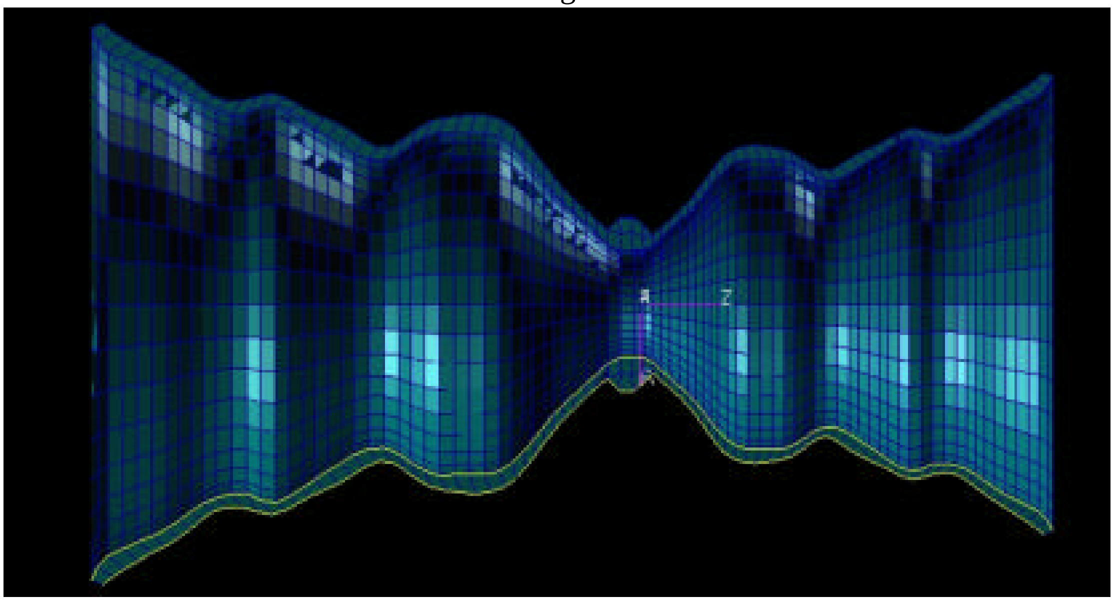
Large bell				Small bell			
mode							
1	2,0	221	1 (A3)	2	2,0	268	1.21 (C#3)
3	2,1	384	1.74 (G4)	4	2,1	429	1.95 (A4)
5	3,0	438	1.98 (of A3)	7	3,0	507	1.89 (of C#3)
9	4,0	655	2.96 (of A3)	12	3,1	908	2.12 (of A4)
11	T3	766					
10	3,1	763	1.99 (of G4)				

Hourbell7 – Optimisation targets for modes shown above:



```
response(strategy=c3,fmin=200)
  mode(var=f1,target=220,tol=1)
  mode(var=f2,target=277,tol=3)
  mode(var=f3,target=392,tol=4)
  mode(var=f4,target=440,tol=4)
  mode(var=f5,target=440,tol=4)
  mode(var=f7,target=554,tol=4)
  mode(var=f9,target=660,tol=4)
  mode(var=f10,target=784,tol=8)
  mode(var=f12,target=880,tol=10)
```

Hourbell8 was a rerun with the same targets to refine the model



Hourbell8 data

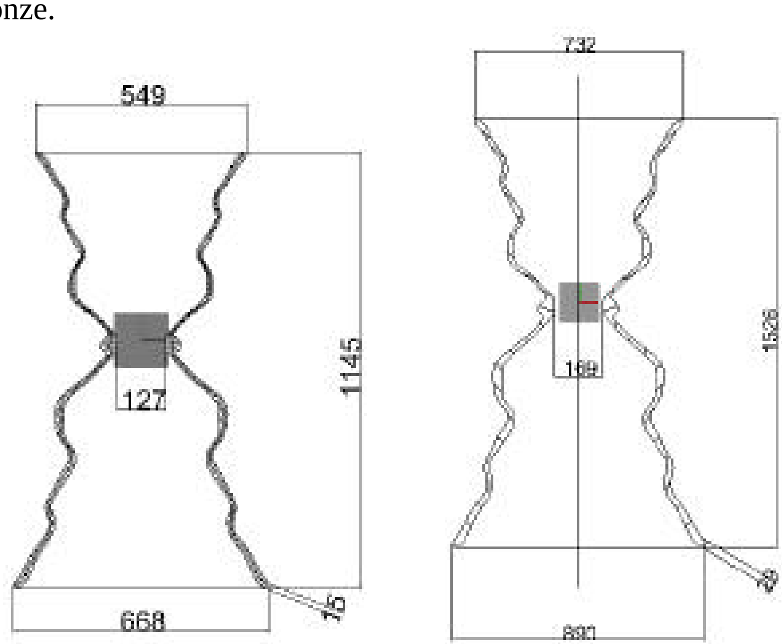
large	mode	Hz	Ratio (Note)	small	mode	Hz	Ratio (Note)
1	2,0	220	1 (A3)	1	2,0	276	1.25 (C#4)
2	2,1	388	1.76 (G4)	2	2,1	437	1.99 (A4)
3	3,0	436	1.97 (of A3)	3	3,0	551	2.06 (of C#4)
4	T3	766	1.99 (of G4)	4	3,1	889	2.03 (of A4)
5	3,1	768	2.00 (of G4)				
6	4,0	656	2.97 (of A3)				

Discussion

The hourbell will produce pitches at A3, C#4, G4 and A4 by the use of a central pivoting rod with offset cams that strikes the bell at one of four locations depending on the direction of motion.

The T3 mode that has been shown in table 2 has been tuned to 1.99 of G4 will not sound when G4 sounds. It will produce a weak secondary tone that is likely to be masked. A further complication is that the manner of the bell support and the joining of the two halves of the hourbell will significantly affect the frequency of this mode (assuming the bell is cast in two halves). This tuning can be adjusted with the cast prototype.

Casting this bell will present a number of technical difficulties. Any geometrical deviations from the profile or porosity in the casting will alter the frequency of modes from the computed results. Computational accuracy can be expected to be within 1%, and it is possible to cast to within 2% of the calculated frequencies. However combined errors can accumulate to require retuning on a lathe. This is further complicated in that in general metal can only be removed in tuning – although some stiffening by welding ribs has been successfully undertaken on bells cast in silicon bronze.



Final profile at original scale and scaled up (left) to produce the pitches E3, G#3, D4